Room temperature high-fidelity non-adiabatic holonomic quantum computation on solid-state spins in Nitrogen-Vacancy centers

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The high-speed implementation and robustness against of non-adiabatic holonomic quantum computation provide a new idea for overcoming the difficulty of quantum system interacting with the environment easily decoherence, which realizing large-scale quantum computer construction. Here, we show that a high-fidelity quantum gates to implement non-adiabatic holonomic quantum computation under electron spin states in Nitrogen-Vacancy(NV) centers, providing an extensible experimental platform that has the potential for room-temperature quantum computing, which has increased attention recent years. Compared with the previous method, we can implement both the one- and two-qubit gates by varying the amplitude and phase of the microwave pulse applied to control the non-Abelian geometric phase acquired by NV centers. We also find that our proposed scheme may be implemented in the current experiment to discuss the gate fidelity with the experimental parameters. Therefore, the scheme adopts a new method to achieve high-fidelity non-adiabatic holonomic quantum computation.

Keywords: non-adiabatic holonomic quantum computation, Nitrogen-Vacancy centers, non-Abelian geometric phase

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I. INTRODUCTION

Recently, the research of the quantum computing [1] has gradually become a hot task. To be useful, quantum computers [2, 3] will require long decoherence time and low error rate. Any quantum computing has to solve the problems including the fault-tolerant features, decoherence and how to prepare high-fidelity quantum gates. Therefore, a method which is particularly useful for controlling errors must be found to implement any kinds of quantum computer. One possible method to achieve more reliable quantum computation is to use geometric phases [4–12] under fluctuations [13, 14] of the path in state space to implement high-fidelity quantum logic gates. Since geometric phases of the total phase obtained by the wave function are divided into adiabatic geometric phases and non-adiabatic geometric phases in different situations, geometric quantum computation is also divided into adiabatic quantum computation [15] and non-adiabatic quantum computation [16–20]. This method, termed holonomic quantum computation eliminates or avoids the low-confidence kinetic phase of the total phase of the wave function in a specific way and makes the quantum gates needed for quantum computation only geometrically. Such a quantum gate only depends on some geometric features, it has a strong fault tolerance and high reliability. Holonomic quantum computation was a general procedure for building universal sets of robust gates using non-Abelian geometric phases which was proposed by *Zanardi* and *Rasetti* in 1999 [21].

Holonomic quantum computation is usually based on adiabatic evolution to achieve quantum computing. However, in adiabatic evolution, the challenge we faced is the adiabatic evolution which needs to fulfill rigorous adiabatic condition. Adiabatic evolution over long periods of operation can make the gate vulnerable to environment-induced decoherence [22–24]. Therefore, the quantum logic gates built in this way usually run extremely slowly. In order to overcome this problem, Sjöqvist *et.al* proposed an optical scheme with a three-level Λ system to achieve non-adiabatic non-Abelian quantum computation which does not require adiabatic conditions [25]. So the gate speed will only depend on the advantages of the quantum system used. Although the method of non-adiabatic holonomic quantum computation has only been proposed recently, it is receiving more and more attention nowadays due to the advantages of non-adiabatic holonomic quantum computation with short running time and robustness to control errors. So far, many theoretical [26–40] and experimental [41–45] proposals have been made to achieve a non-adiabatic holonomic gates.

In this paper, we demonstrate an experimentally fea-

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sible non-adiabatic optical transitions scheme to implement the arbitrary one- and two-qubit logical gates by using electron spin states in NV centers under room temperature. The NV centers [42, 44] system is considered to be the best candidate for quantum computing in physical systems because of its long enough electron spin lifetime and relatively good coherence control even at room temperature. Therefore, our scheme not only avoids long-time requirements, but also retains complete robustness and reduces exposure time to various sources of error. It is especially useful when evolutionary time is shorter than coherence time. In addition, numerical simulations were performed by using experimental physical parameters, we further find that the quantum gate we proposed has a high-fidelity in a non-adiabatic environment. It means that our theoretical scheme may be completed experimentally, which also provides people a common and powerful solid-state quantum computing method.

II. NON-ADIABATIC HOLONOMIC QUANTUM COMPUTATION

In previous research we know that the universal quantum gate is geometrically constructed without the need of adiabatic conditions, thus combining speed with universality. Here, before we further introduce our physical model, let us start with a briefly introduction to how the non-adiabatic holonomy appears in unitary evolution [25, 36]. We consider use an N-dimensional Hilbert space to describe a quantum system whose Hamiltonian is $H(t)$. Suppose there is a time-dependent N-dimensional subspace $M(0)$, which consists of a set of orthogonal basis vectors $\{|\phi_k(t)\rangle\}_{k=1}^{N}$ and the initial state of the system is in this subspace. Here, $|\phi_k(t)\rangle$ satisfy the Schrödinger equation $i|\partial\phi_k(t)\rangle = H(t)|\phi_k(t)\rangle$. Among them, $|\phi_k(0)\rangle \to |\phi_k(t)\rangle = U(t,0)|\phi_k(0)\rangle$ and $U(t,0) = Texp(-i\int_0^t H(t')dt')$ with T being time ordering. The unitary transformation is a non-adiabatic holonomy matrix acting on the subspace $M(0)$ if the following two conditions are satisfied: (i) $\sum_{k=1}^{N}|\phi_k(\tau)\rangle\langle\phi_k(\tau)| = \sum_{k=1}^{N}|\phi_k(0)\rangle\langle\phi_k(0)|$ and (ii) $\langle\phi_k(t)|H(t)|\phi_l(t)\rangle = 0$, $l = 1, 2, ...L$. Here, the first condition guarantees that the evolution of subspace $S(0)$ is cyclic; the second condition ensures that the dynamical phase disappears and evolution is purely geometric.

III. ONE-QUBIT GATE

Here, we will construct the one-qubit geometric gates physical model by manipulating the electron spin states of NV center at room temperature. The high degree of electronic confinement in NV does help their coherence by keeping the wavefunctions small, but that is true for any sort of defect or donor-based electron trapping. The reason for the high coherence times in diamond is more because diamond has very low spin-orbit coupling and the vast majority of carbon isotopes have no nuclear spin. This creates a very low noise magnetic background for the NV center spins, so the NV centers will be able to avoid interference from the external environment, this is the main reason why for the better coherence. The most significant feature of the NV center spins are typically initialized and measured with optical pumping but are controlled with microwave pulses at room temperature. Based on this, NV center is an excellent qubit carrier. The related level of the solid-state spin of the NV center can be regarded as a V-type configuration as shown in FIG. 1(a). The ground state ${}^3A_2$ of the NV center is the spin triplet state. There is a $D = 2\pi \times 2.87GHz$ zero-field splitting between $m_s = 0$ and $m_s = \pm 1$. We take the Zeeman components $|m = -1\rangle \equiv |0\rangle$ and $|m = +1\rangle \equiv |1\rangle$ as the qubit basis states and use$|m = 0\rangle \equiv |e\rangle$ as an ancillary level for geometric manipulation of the qubit. During the holonomic transformation, the $|e\rangle$ state remains unoccupied before and after the quantum gate just as an idle ancilla. In the rotating-wave approximation and the interaction picture, the Hamiltonian of the system is (In this paper, we choose $\hbar = 1$)

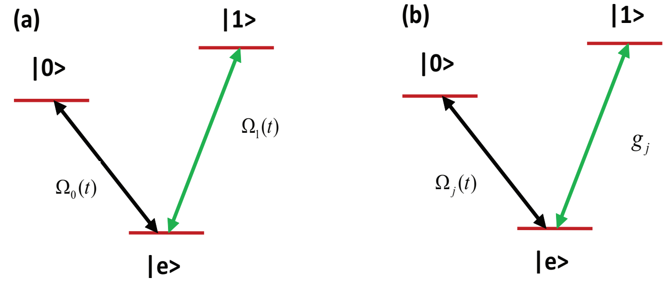

FIG. 1: (a) Setup for non-adiabatic holonomic one-qubit gate in the spin-triplet ground state of the NV center and the microwave coupling configuration. (b) Coupling configuration of two NV center by microwave pulse with Rabi frequency $\Omega_j(t)$ and couples to the cavity with strength g_j.

$$H(t) = \Omega_0(t)|0\rangle\langle e| + \Omega_1(t)|1\rangle\langle e| + H.c. \tag{1}$$

Here, we assume that the Rabi frequencies $\Omega_0(t) = \Omega(t)\cos(\theta/2)$ and $\Omega_1(t) = \Omega(t)\sin(\theta/2)$, where $\Omega(t)$ is the real-valued envelope and θ is a time-independent parameter representing the relative strengths of the two Rabi frequencies. So we can easily know $\Omega_1(t)/\Omega_0(t) = \tan(\theta/2)$. We vary the amplitude $\Omega(t) = \sqrt{\Omega_0^2(t) + \Omega_1^2(t)}$, and we can get the eigenstates of the this system as follows

$$
\begin{aligned}
|D_0\rangle &= \sin(\theta/2)|0\rangle - \cos(\theta/2)|1\rangle \\
|D_-\rangle &= [\cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle - |e\rangle]/\sqrt{2} \\
|D_+\rangle &= [\cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle + |e\rangle]/\sqrt{2}.
\end{aligned} \tag{2}
$$

As is shown in this result, the effective Hamiltonian can be rewritten as

$$
H_I(t) = \Omega(t)(|b_\theta\rangle\langle e| + |e\rangle\langle b_\theta|). \tag{3}
$$

Where $|b_\theta\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle$ is the bright state, the state orthogonal to $|b_\theta\rangle$ will be denoted as $|d_\theta\rangle = \sin(\theta/2)|0\rangle - \cos(\theta/2)|1\rangle$, $\{|d_\theta\rangle, |b_\theta\rangle\}$ spans the same subspace as that by $\{|0\rangle, |1\rangle\}$. In this dressed-state representation, the states $|d_\theta\rangle$ and $|e\rangle$ are only coupled by the dynamics, while the dark state $|d_\theta\rangle = \sin(\theta/2)|0\rangle - \cos(\theta/2)|1\rangle$ decouples from the dynamics all the time. It follows that the qubit subspace $M(0)$ evolves into $M(t)$ spanned by $|\varphi_k(t)\rangle = Texp(-i\int_0^t H_I(t')dt')|k\rangle = U(t,0)|k\rangle$, $k = 0, 1$, which undergoes cyclic evolution if the pulse pair satisfies $\int_0^{\tau_1} \Omega(t')dt' = \pi$. In contrast to adiabatic schemes, the $|\varphi_k(t)\rangle$ are not instantaneous eigenstates of $H_I(t)$. Here, the Rabi weights $\sin(\theta/2)$ and $\cos(\theta/2)$ need to be properly normalized $|\sin(\theta/2)| + |\cos(\theta/2)| = 1$. To ensure the parallel transport condition for a purely geometric evolution $\langle\varphi_k|H_I(t)|\varphi_j\rangle = \langle k|H_I(t)|j\rangle = 0$ the ratio of the Rabi frequencies $[\sin(\theta/2)/\cos(\theta/2)]$ has to be kept constant. Under the above conditions, the final time evolution operator $U_1(\theta)$ projected onto the computational space spanned by $\{|0\rangle, |1\rangle\}$ defines the holonomic one-qubit gate

$$
U_1(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \tag{4}
$$

where θ can be controlled by adjusting the amplitude and relative phase of the microwave pulse so that a set of non-adiabatic universal single qubit gates can be realized. For example, when $\theta = \pi/4$, a Hadamard gate is realized. Meanwhile one can combine $\theta = \pi/2$ to implement a NOT gate.

IV. NUMERICAL SIMULATION AND DISCUSSIONS

Dephasing is caused by the inevitable interaction of the system with its environment. For a variety of systems, dynamics is the major source of decoherence. We suppose that the Markovian approximation is valid for the system and the effect of dephasing can be described by the Lindblad equation

$$
\frac{\partial\rho}{\partial t} = -i[H_I, \rho] + [\gamma_y L(A^-) + \gamma_x L(S^-) + \gamma_z L(S^z)]/2 \tag{5}
$$

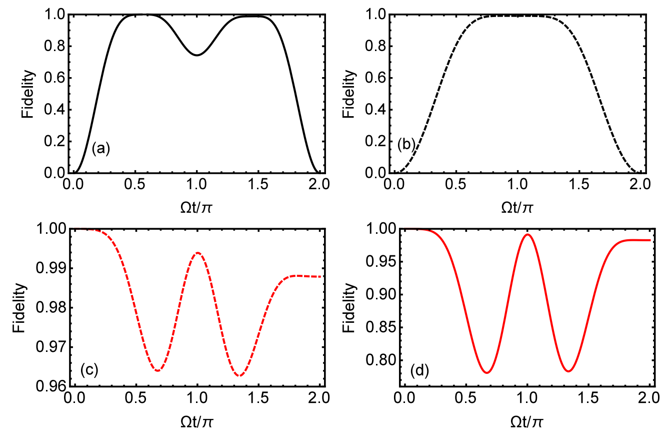

FIG. 2: Numerical simulation results for single-qubit geometric gates with the initial states be taken as $|0\rangle\{|1\rangle\}$ State fidelities for $(a)\{(c)\}$ the Hadamard gate$(\theta = (\theta/4))$ and $(b)\{(d)\}$ the NOT gate$(\theta = (\theta/2))$.

Where ρ is the density matrix operator for the hybrid systems, $L(A) = 2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A$ is the Lindblad operator. In order to achieve our purpose, we take Hadamard gate and NOT gate as typical examples. The parameters that we choose to experimentally implement are as follows: the Rabi frequency $\Omega = 2\pi \times 300MHz$, the qubit relaxation and dephasing rates can be chosen as: $\gamma_y \approx 2\pi \times 5kHz$, $\gamma_x = \gamma_z \approx 2\pi \times 1.5MHz$ [26, 46, 47]. Here, Fig. 2(a) and 2(b) show the fidelity of the Hadamard gate and the NOT gate over time when the initial state of the qubit is $|0\rangle$ and the cavity is under vacuum. We find that the maximum fidelity can reach 99.95% and 99.93%, respectively. In Fig. 2(c) and 2(d), we plot the Hadamard gate and the NOT gate with the initial state of the qubit in the $|1\rangle$ state fidelity curve, the maximum fidelity at this time were 98.83% and 98.20%, respectively. In addition, for a superposition initial state $|\varphi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ corresponding to the ground state, our numerical calculation the gate fidelities of Hadamard and Not gates can reach 97.20% and 99.85% as shown in Fig. 3.

V. TWO-QUBIT GATE

Next we set out to achieve two-qubit gate with the ability to suppress systematic errors that require a coupling between two NV centers. We consider the hybrid quantum system, in which two NV centers are coupled to microwave resonator, as shown in Figure 1(b). The NV centers can be modeled as a V-style three-level qubit with $|0\rangle$ and $|1\rangle$ being two upper levels, and $|e\rangle$ serving as the lower level. $\Omega_j(t)$ is the Rabi frequency of microwave pulse driving the transition $|e\rangle \leftrightarrow |0\rangle$, g_j is the coupling strength between the NV centers and the cavity mode. By using the rotating frame and the rotating wave approximation, the interaction Hamiltonian can be written

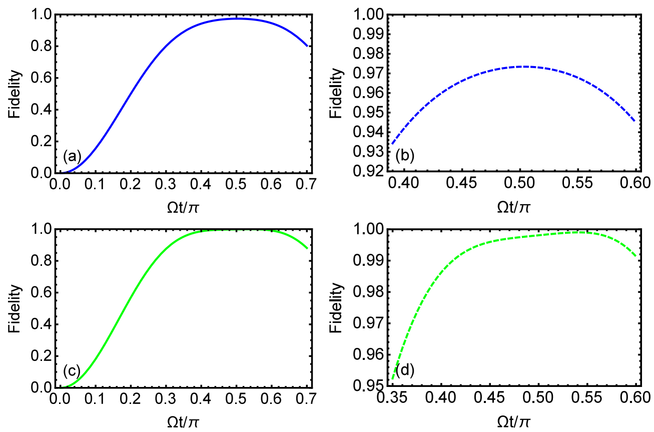

FIG. 3: Numerical simulation results for single-qubit geometric gates with the initial states be taken as $|\varphi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. State fidelities for (a) the Hadamard gate and (c) the NOT gate. Figure.3$(b)\{(d)\}$ is a part of Figure.3$(a)\{(c)\}$, We can get the maximum fidelity of the Hadamard(NOT) gate very accurately from Figure. 3$(b)\{(d)\}$.

as

$$H_{\rm eff}(t) = \eta_1(t)\alpha|1_1\rangle\langle 0_1| + \eta_2(t)\alpha|1_2\rangle\langle 0_2| + H.c \qquad (6)$$

Here, $\eta_j(t) = g_j\Omega_j(t)(j = 1, 2)$ is the effective coupling strength. It can be adjusted according to the amplitude of the corresponding externally driven microwave pulse. The whole system evolves in the one-excited subspace spanned by $\{|\Psi_1\rangle = |00_1 1_2\rangle, |\Psi_2\rangle = |01_1 0_2\rangle, |\Psi_3\rangle = |10_1 0_2\rangle\}_{c,1,2}$ where the subscripts c, 1 and 2 donate the cavity mode, the first NV center, and the second NV center, respectively. For this scheme, the fidelity of the two-qubit gate is defined as $F = \langle\Psi_1|\rho|\Psi_1\rangle$ with $|\Psi_1\rangle$ being the corresponding ideally final state under the population transfer on its initial state $|\Psi_2\rangle$. The effective Rabi frequency $\lambda = \sqrt{\eta_1^2(t) + \eta_2^2(t)}$, and the ratio $|\eta_1^2(t)|^2/|\eta_2^2(t)|^2 = \tan(\vartheta/2)$ should be kept as a constant during each pulse pair. It implies that $exp[-i\int_0^{\tau_2} H_{\rm eff}(t)dt]$ under the π pulse criterion $\lambda\tau_2 = \pi$. Therefore, the nontrivially holonomic two-qubit gate in the subspace $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ reads as

$$U_2(\vartheta) = \begin{pmatrix} \cos\vartheta & \sin\vartheta & 0 & 0 \\ \sin\vartheta & -\cos\vartheta & 0 & 0 \\ 0 & 0 & -\cos\vartheta & \sin\vartheta \\ 0 & 0 & \sin\vartheta & \cos\vartheta \end{pmatrix} \qquad (7)$$

Obviously, one can implement nontrivial two-qubit holonomic logical gate by controlling the ϑ. If the initial state is in $|10\rangle$, we can simulate the state population of quantum states and fidelity of the two-qubit gate $U_2(\vartheta = \pi/4)$, which is shown in Fig. 4, the fidelity can reach 99.94%. In our numerical simulation, we choose the feasible experimental parameters as $\Omega_j/\sqrt{2} = g_j = 2\pi \times 1GHz$ and $\kappa = 2\pi \times 56kHz$ [26, 48].

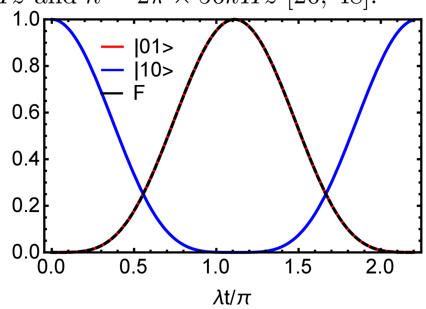

FIG. 4: State populations and fidelity for the two-qubit gate $U_2(\vartheta = \pi/4)$ with the initial state being $|10\rangle$.

VI. CONCLUSION

In summary, we proposed a universal non-adiabatic quantum computation scheme by manipulating the NV centers electron spins that paves the way for all-geometry quantum computation in solid-state systems. By controlling the amplitude and relative phase of the microwave pulses, a universal set of one-qubit and two-qubit gates are constructed on the encoded logical qubits. Among them, the one-qubit gates can be achieved through external microwave-driving fields, and the two-qubit gates can be obtained in a fast resonant way. Our result shows that the robust high-fidelity holonomic phase gates by using experimental parameters. Therefore, it adopts a new method for a quantum computer to build holonomic quantum processors. Moreover, the holonomic quantum gate can also be used for the optimization or calibration of the transitions between four Bell-states [49] and the transfer of quantum states over long distances to build holonomic quantum repeater for long-distance quantum communication networks [45].

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